Term-structure analysis of hidden order in the limit order book: evidence from the E-Mini S&P 500

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Outline

- Background & Motivation
- Related Literature
- Methodology
- Results and analysis
- Conclusions
Background & Motivation

Hidden Order

- **Hidden order** is currently **increasingly popular** as a standard feature of electronic limit order book markets.

- **Iceberg Order, Invisible Order, Invisible Liquidity, Hidden Order.**

- **Max Show** on CME’s Globex Electronic Platform.

- Hidden order allows traders to **hide all or partially hide** their orders, they can divide their large orders into smaller parts so that the public sees only a small portion of the order at a time.

- **By hiding its large size**, the hidden order **reduces the price movements** caused by substantial changes in a stock’s supply and demand **to avoid exposure to risk.**
Although market participants who use hidden orders commonly lose time priority to traders who submit displayed orders, this time disadvantage is offset by the secrecy afforded by the hidden orders strategy.

However, the more serious cost is that because of the time lag, some hidden orders are not possible to execute.

The advantage of entirely or partially hidden orders is that it reduces the risk of being undercut by aggressive or high-speed traders and this potential loss is always greater than the cost of losses though time priority, especially for agents who want to submit a large order.

Clearly, hidden orders have both costs and benefits compared to visible orders.
Objective of this talk

- For this presentation we will look specifically at **VWAPTD-Hidden order detection algorithm** and we will apply this to trading in the E-Mini S&P 500 futures contract.
- We estimate the daily fraction of hidden order for complete transaction for the E-Mini between 2008 and 2015.
- We will then set up a series of experiments to see the association between hidden order and our observed market environment proxies.
Executive Summary

- Our algorithm show **43%** all of trade volume in the E-Mini S&P 500 is involved with invisible liquidity.

- We also find that **price impact is decreased and market quality is improved** with the presence of hidden order both during high and low frequency trading periods.

- We use this measure to study the association between hidden order and other observed market environments. **Our analysis finds aggressive hidden order activity when trading volume is increased.**
Contribution

- **First**, this is the only research study to provide a comprehensive innovation of signed-hidden order detection algorithm for E-min S&P500 for limit order book data.

- **Second**, to implement the detection algorithm, this analysis applies the Volume-Weighted Average Price (VWAP) approach with the signed-trade direction indicator to introduce the Volume Weighted Average Price-Trade Direction (VWAPTD) indicator.

- **Third**, the empirical application of a term-structure analysis is a new contribution to the literature in the field, and my algorithms are available for other researchers to implement in such studies in a different market setting.

- **Lastly**, the advantage of the algorithm is that it can be constructed from publicly available data, therefore, it does not rely on special data.
Instances of hidden liquidity in financial markets are increasingly popular, and we provide some evidence in the following section.

- Impact of hidden order to the market; Aitken et al. (2001, JFM) and De Winne and D’hondt (2007, RF).
- Trading costs and trade time; Bessembinder et al. (2009, JFE).
- Order size and price impact; Frey and Sandås (2009, AFA Meeting).
- However, Pardo and Pascual (2012, JF) show that there is no significant relationship between hidden order and price impact.
- To understand the behavior of traders who submit hidden liquidity; Gozluklu et al. (2009, EFA Meeting) and Bloomfield et al. (2015, JF)
For the last decade, the financial markets environment has been different in fundamental ways. Speed is one of the most important factors due to information gathering and the actions prompted by this information have created high volatility in the market (Hasbrouck and Saar (2013)).

- In comparison, High-frequency trading evolution respond at a pace 100 times faster than it would take for a human trader to blink.
- The fastest trade updated for Eurodollar futures is 500 microseconds and fastest trade updated for E-Mini S&P 500 futures is 60 microseconds.
- The average trade updated is 45 and 2.5 seconds for Eurodollar and E-Mini S&P 500 respectively.

At these speeds, only the microstructure matters (O’Hara (2015)).
The data is recorded in millisecond time stamp


The raw tapes were streamed into a new format ‘hdf5’ which provided a high-integrity medium for this amount of data.

The E-Mini S&P 500 data was reduced to 143 GB of compressed hdf5 data stored in separate files by maturity date, and then stored on a solid state drive.

In total, the data prepared for each data set was around four months, from downloading the raw data from TRTH to data cleaning procedures.
### Ex-LOB & HFT Data

#### Table 1.1: The Limit Order Book data of the E-Mini S&P 500 on May 05, 2013 (ESU3), time between 22:43:06.283 Hours and 22:43:18.809 Hours

<table>
<thead>
<tr>
<th>Time</th>
<th>Ask Price</th>
<th>Volume</th>
<th>Number of Traders</th>
</tr>
</thead>
<tbody>
<tr>
<td>05May13</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22:43:06.283</td>
<td>1600.75</td>
<td>1606.25</td>
<td>1606.5</td>
</tr>
<tr>
<td>22:43:06.283</td>
<td>1600.75</td>
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<td>22:43:06.283</td>
<td>1600.75</td>
<td>1606.25</td>
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<table>
<thead>
<tr>
<th>Time</th>
<th>Bid Price</th>
<th>Volume</th>
<th>Number of Traders</th>
</tr>
</thead>
<tbody>
<tr>
<td>05May13</td>
<td></td>
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<td>1600.25</td>
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<td>22:43:06.283</td>
<td>1600.25</td>
<td>1606.25</td>
<td>1606.5</td>
</tr>
</tbody>
</table>

PP & JW

ES-Hidden Order

February 2, 2017
HFT data as a “Big Data”

- The differences between small, medium, and Big data (M. E Driscoll(2010))

<table>
<thead>
<tr>
<th>class</th>
<th>size</th>
<th>manage with</th>
<th>how it fits</th>
<th>examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>small</td>
<td>&lt; 10 GB</td>
<td>Excel, R</td>
<td>fits in one machine’s memory</td>
<td>thousands of sales figures</td>
</tr>
<tr>
<td>medium</td>
<td>10GB-1TB</td>
<td>indexed files, monolithic DB</td>
<td>fits on one machine’s disk</td>
<td>millions of web pages</td>
</tr>
<tr>
<td>Big</td>
<td>&gt; 1TB</td>
<td>Hadoop, distributed DBs</td>
<td>stored across many machines</td>
<td>billions of web clicks</td>
</tr>
</tbody>
</table>

- Manyika et al. (2011) explain big data is the data that can create significant value for the world economy, enhancing the productivity and competitiveness of companies and the public sector and creating substantial economic surplus for consumers.
Ex. LOB & HFT Data of E-Mini S&P500 on Sep 6, 2013

The Limit Order Book Prices of E-Mini S&P500 on September 6, 2013 (ESU3)

The Accumulated Volume in Limit Order Book of E-Mini S&P500 on September 6, 2013 (ESU3)

The Accumulated Number of Trader in Limit Order Book Volume of E-Mini S&P500 on September 6, 2013 (ESU3)
The Sample: E-Mini S&P 500 Futures Contract

This project utilizes the HFT comprehensive data set for trade, quote and limit order book (LOB) pulled from Thomson Reuters Tick History (TRTH) taped in millisecond time stamp.

- The E-Mini S&P 500 (ticker symbol: ES), is a stock market index futures contract.
- The E-mini traded exclusively on the CME Globex trading platform in a fully electronic limit order market, 24 hours a day from Sunday to Friday 17.00 - 16.00 (Chicago Time/CT) with 15-minute technical maintenance break each day.
- It is largest trading volume in the CME.
- This work uses publicly available data from CME's Globex collected from TRTH.
- This data set includes 32 E-Mini contracts from July, 2 2008 until June 19, 2015.
ES-Data Cleaning Procedures

The cleaning procedure of this data is carried out following steps S1-S5. This step is applied to all trades and limit order book data.

S 1: Delete entries quotes with bids (offers) that are greater (smaller) than offers (bids) or mis-priced from trades and quotes data.

S 2: Delete entries a bid, ask or trade volume equal to zero and NaN value. This technique is to eliminate nonessential values in this time series data.

S 3: Delete duplicate data from both trades and quotes data. This organization and sorting technique helps to reduce costs and time, and improve efficiency in the analysis process.

S 4: Retain entries missing values and match with equivalent time vector for each variable.

S 5: Finally, trades and standing quotes is matched at the time of the trade time stamp.
Detecting Hidden Orders

S 1: First, the limit order book (LOB) and trade files are matched.

S 2: Next, trade is classified as a buy or sell by using volume weighted average price trade direction (VWAPTD) with +1 for a buy trade and -1 for a sell trade.

S 3: Then, if the trade is classified as a buy then the algorithm compares the trade reported size with the corresponding updates volume (changes in the accumulated volume) on bid side in the LOB.

S 4: If the trade is classified as a sell, then the algorithm compares the trading volume with the corresponding updates volume on the ask side.

S 5: To infer the volume of hidden order, the trade volume is compared with the volume update in LOB. If the trade size is larger than the corresponding updates volume, a deviation between these two volumes can only be explained by the presence of invisible or a hidden volume.

S 6: However, for this algorithm, if the corresponding updates volume in LOB is positive or larger than the reported trade size, the algorithm classifies this as a modification order.

<table>
<thead>
<tr>
<th>Time</th>
<th>B/S</th>
<th>Price</th>
<th>Volume</th>
<th>Volume</th>
<th>Bid</th>
<th>Offer</th>
<th>Bid</th>
<th>Offer</th>
</tr>
</thead>
<tbody>
<tr>
<td>15:49:33.706</td>
<td>Buy</td>
<td>1914.75</td>
<td>2</td>
<td></td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15:49:33.707</td>
<td>Buy</td>
<td>1914.75</td>
<td>3</td>
<td></td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15:49:33.707</td>
<td>Sell</td>
<td>1914.75</td>
<td>1</td>
<td></td>
<td>63</td>
<td>-8</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
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<td>Sell</td>
<td>1914.75</td>
<td>5</td>
<td></td>
<td>20</td>
<td>-250</td>
<td>0</td>
<td>255</td>
</tr>
<tr>
<td>15:49:33.707</td>
<td>Sell</td>
<td>1914.75</td>
<td>50</td>
<td></td>
<td>131</td>
<td>-102</td>
<td>0</td>
<td>52</td>
</tr>
<tr>
<td>15:49:33.718</td>
<td>Sell</td>
<td>1914.75</td>
<td>1</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15:49:33.770</td>
<td>Sell</td>
<td>1914.75</td>
<td>1</td>
<td></td>
<td>2</td>
<td>-3</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>15:49:33.770</td>
<td>Buy</td>
<td>1915.00</td>
<td>2</td>
<td></td>
<td>-8</td>
<td>0</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>15:49:33.770</td>
<td>Sell</td>
<td>1914.75</td>
<td>1</td>
<td></td>
<td>13</td>
<td>-20</td>
<td>0</td>
<td>19</td>
</tr>
<tr>
<td>15:49:33.804</td>
<td>Sell</td>
<td>1914.75</td>
<td>1</td>
<td></td>
<td>27</td>
<td>-2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>15:49:33.900</td>
<td>Sell</td>
<td>1914.75</td>
<td>1</td>
<td></td>
<td>31</td>
<td>-33</td>
<td>0</td>
<td>32</td>
</tr>
<tr>
<td>15:49:33.900</td>
<td>Sell</td>
<td>1914.75</td>
<td>2</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: This table presents an example of the procedure of the hidden order detection algorithm that was taken from the activity in the E-mini futures on August 10, 2014 (ESU4). The table reports order activity starting around 15:49:33.706 pm and ending at 15:49:33.900 am. Shading identifies hidden order volume corresponding between 1 and 255 volumes for bid and offer. In total, during this roughly 300 millisecond period, there are 9 volumes of hidden order from the buyer and 370 volumes from the seller.

Note: This figure presents the number of hidden orders on the E-Mini S&P 500 from 2008 to 2015. The upper sub-figure presents a daily number of hidden orders and the lower sub-figure presents an average number by week from eighteen to one week to maturity.
To illustrate VWAP for bid and ask price, we give the following example. Assume that at time $t$ on a trading day, traders submit sell orders in a limit order book. For level 1 (L1) is 5 volume, L2 is 10, L3 is 12, L4 is 12, L5 is 10 and the price is 98, 98.5, 99, 99.2 and 99.5 for L1 to L5 respectively. Thus, the VWAP for sell ($VWAP_{ask,t}$) is equal to:

$$\frac{(98 \times 5) + (98.5 \times 10) + (99 \times 12) + (99.2 \times 12) + (99.5 \times 10)}{5 + 10 + 12 + 12 + 10} = 98.95$$

For buy orders in limit order book at level 1 (L1) it is 6 volume, L2 is 10, L3 is 13, L4 is 12, L5 is 10 and the price is 97, 96.5, 96, 95.5 and 95 for L1 to L5 respectively. Thus, the VWAP for buy ($VWAP_{bid,t}$) is equal to:

$$\frac{(97 \times 6) + (96.5 \times 10) + (96 \times 13) + (95.5 \times 12) + (95 \times 10)}{6 + 10 + 13 + 12 + 10} = 96.84$$
This work uses $VWAP_{bid_t}$ and $VWAP_{ask_t}$ to calculate mid price at time $t$ equal to:

$$mid_t = \frac{VWAP_{ask_t} - VWAP_{bid_t}}{2}.$$  

After calculating the mid point by using VWAP, tick rule is applied for trade classification. The tick rule assumes that trades are buys if the trade price is higher than the previous one; on the other hand, if the trade price is lower than the previous one, it is assumed that the trade is a sell. If the trade price remains stable compared to the previous price, the trade is assumed to be the same as the previous trade.
Market Quality Proxy

- **Quoted spread:**
  \[
  Spread_t = \frac{VWAP_{ask_t} - VWAP_{bid_t}}{mid_t}
  \]  
  \[ (2) \]

- **Effective spread:**
  \[
  EffSpread_t = \frac{TD_t(p_t - mid_t)}{mid_t}.
  \]  
  \[ (3) \]

- **Realized spread:**
  \[
  RSpread_t = \frac{TD_t(p_t - mid_{t+1})}{mid_t}.
  \]  
  \[ (4) \]

- **Price impact:**
  \[
  PImp_{t} = \frac{TD_t(mid_{t+1} - mid_t)}{mid_t}.
  \]  
  \[ (5) \]

where \( TD_t \) is signed-trade direction indicator that equals 1 for buy-initiated and -1 for sell-initiated.
To gauge the effect of the hidden order to market liquidity and market quality, I adapt the OLS from Hasbrouck and Saar (2013) with our hidden order study.

- The first model is:

\[ MktQuality_{i,t} = a_1 MarketConcentration_{i,t} + a_2 HiddenOrderIntensity_{i,t} + \nu_{i,t} \]  
(6)

- the second model as:

\[ MktQuality_{i,t} = a_1 MarketConcentration_{i,t} + a_2 AskHDC_{i,t} + a_3 BidHDC_{i,t} + \nu_{i,t} \]  
(7)
The Vector Auto-Regression Model (VAR)

This VAR system in this work can be defined as: the quote revision equation

\[
\tilde{QR}_t = \sum_{i=1}^{5} \alpha_{1,i} \tilde{QR}_{t-i} + \sum_{i=1}^{5} \alpha_{2,i} \tilde{TD}_{t-i} + \sum_{i=1}^{5} \alpha_{3,i} \tilde{HDV}_{t-i} + \sum_{i=1}^{5} \alpha_{4,i} \tilde{TV}_{t-i} + \nu_t^{qr}.
\]  

(8)

The signed-trade direction equation

\[
\tilde{TD}_t = \sum_{i=1}^{5} \alpha_{1,i} \tilde{QR}_{t-i} + \sum_{i=1}^{5} \alpha_{2,i} \tilde{TD}_{t-i} + \sum_{i=1}^{5} \alpha_{3,i} \tilde{HDV}_{t-i} + \sum_{i=1}^{5} \alpha_{4,i} \tilde{TV}_{t-i} + \nu_t^{td}.
\]  

(9)

The signed-hidden order volume equation

\[
\tilde{HDV}_t = \sum_{i=1}^{5} \alpha_{1,i} \tilde{QR}_{t-i} + \sum_{i=1}^{5} \alpha_{2,i} \tilde{TD}_{t-i} + \sum_{i=1}^{5} \alpha_{3,i} \tilde{HDV}_{t-i} + \sum_{i=1}^{5} \alpha_{4,i} \tilde{TV}_{t-i} + \nu_t^{hdv}
\]  

(10)

The signed-trade volume equation

\[
\tilde{TV}_t = \sum_{i=1}^{5} \alpha_{1,i} \tilde{QR}_{t-i} + \sum_{i=1}^{5} \alpha_{2,i} \tilde{TD}_{t-i} + \sum_{i=1}^{5} \alpha_{3,i} \tilde{HDV}_{t-i} + \sum_{i=1}^{5} \alpha_{4,i} \tilde{TV}_{t-i} + \nu_t^{tv}.
\]  

(11)

Where the \( \tilde{QR}_t \) is quote revision at time \( t \), \( \tilde{QR}_t = 100 \times (\ln mid_t - \ln mid_{t-1}) \), \( \tilde{TD}_t \) is the signed-trade direction indicator at time \( t \), \( \tilde{HDV} \) is signed-hidden order volume and \( \tilde{TV} \) is signed-trading volume at time \( t \), \( \nu_t \) is white noise which may be correlated with a variance-covariance matrix \( \Sigma \).
The empirical study begins by systematizing E-mini S&P500 time series data in the form of the term-structure. After this,

- The E-mini data is separated into 18 sets of data, which is from eighteen to one week to expiration.
- I find the E-mini trading becomes highly active from fourteen weeks until two weeks to maturity.
- From fifteen to fourteen weeks, the trade updated jumps from 0.64 to 2.16 million updated, results for a period longer than fifteen weeks to maturity, showing that trade updated declines continuously.
- The sixteen, seventeen and eighteen weeks to maturity, the average trade updated per week is 0.019, 0.008, and 0.005 million updated respectively.
Cumulative differences of median price impacts of hidden order from buyer and seller.

Note: This figure presents the median percentage change in cumulative price impact beginning from 20 steps prior to selling hidden order (red thick line), buy hidden order (blue thick line) and ordinary order (* line) occur in the E-mini S&P 500 market. The figure also shows the standard errors (dashed - - line). This figure, additionally, presents overlay plots of the cumulative differences of median price impacts of hidden order from eighteen to one week's data samples from 30 E-mini S&P 500 (dotted · · · line).
Hidden Order & Market Quality: OLS estimates (Model 1)

Model 1:
$$MktQuality_{i,t} = a_1 MarketConcentration_{i,t} + a_2 HiddenOrderIntensity_{i,t} + \nu_{i,t}$$

<table>
<thead>
<tr>
<th>Spread</th>
<th>Week_01</th>
<th>Week_06</th>
<th>Week_12</th>
<th>Week_18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coeff.</td>
<td>0.095</td>
<td>0.091</td>
<td>0.500</td>
<td>0.474</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.213)</td>
<td>(&lt;0.001)**</td>
<td>(&lt;0.001)**</td>
<td>(&lt;0.001)**</td>
</tr>
<tr>
<td>R²</td>
<td>(0.022)</td>
<td>(0.208)</td>
<td>(0.210)</td>
<td>(0.234)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>EffSpreadCoff.</th>
<th>Week_01</th>
<th>Week_06</th>
<th>Week_12</th>
<th>Week_18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coeff.</td>
<td>0.674</td>
<td>-0.2600.030</td>
<td>-0.3930.088</td>
<td>-0.4050.887</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(&lt;0.001)**</td>
<td>(&lt;0.001)**</td>
<td>(&lt;0.001)**</td>
<td>(&lt;0.001)**</td>
</tr>
<tr>
<td>R²</td>
<td>(0.411)</td>
<td>(0.166)</td>
<td>(0.184)</td>
<td>(0.804)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ReSpread</th>
<th>Week_01</th>
<th>Week_06</th>
<th>Week_12</th>
<th>Week_18</th>
</tr>
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<tbody>
<tr>
<td>Coeff.</td>
<td>0.663</td>
<td>-0.2140.006</td>
<td>-0.3800.071</td>
<td>-0.3740.811</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(&lt;0.001)**</td>
<td>(&lt;0.001)**</td>
<td>(&lt;0.001)**</td>
<td>(&lt;0.001)**</td>
</tr>
<tr>
<td>R²</td>
<td>(0.393)</td>
<td>(0.146)</td>
<td>(0.154)</td>
<td>(0.625)</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Pimpt</th>
<th>Week_01</th>
<th>Week_06</th>
<th>Week_12</th>
<th>Week_18</th>
</tr>
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<tbody>
<tr>
<td>Coeff.</td>
<td>0.439</td>
<td>-0.3850.190</td>
<td>-0.3610.238</td>
<td>-0.5790.784</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(&lt;0.001)**</td>
<td>(&lt;0.001)**</td>
<td>(&lt;0.001)**</td>
<td>(&lt;0.001)**</td>
</tr>
<tr>
<td>R²</td>
<td>(0.239)</td>
<td>(0.237)</td>
<td>(0.465)</td>
<td>(0.702)</td>
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</tbody>
</table>

The hidden liquidity measure is $HiddenOrderIntensity_{i,t}$, this is a fraction of hidden order volume and trades volume. $MktQuality_{i,t}$ is a placeholder denoting: Spread is quoted spread; EffSpread is the effective spread; ReSpread is the realized spread; Pimpt is the price impact. MarketConcentration is the function of a number of traders and their respective volume of the total quote volume in LOB.
Hidden Order & Market Quality: OLS estimates (Model 2)

Model 2: \( MktQuality_{i,t} = a_1 MarketConcentration_{i,t} + a_2 AskHDC_{i,t} + a_3 BidHDC_{i,t} + \nu_{i,t} \)

<table>
<thead>
<tr>
<th></th>
<th>Week_01</th>
<th></th>
<th></th>
<th>Week_06</th>
<th></th>
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<th>Week_12</th>
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<th>Week_18</th>
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<tbody>
<tr>
<td><strong>Spread</strong></td>
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<tr>
<td>Coffee</td>
<td>0.092</td>
<td>0.081</td>
<td>0.155</td>
<td>0.061</td>
<td>0.330</td>
<td>0.586</td>
<td>0.263</td>
<td>0.674</td>
<td>0.778</td>
<td>0.556</td>
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<tr>
<td>( p-value )</td>
<td>(0.378)</td>
<td>(0.208)</td>
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<td>(&lt;0.001)**</td>
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<td>(0.053)</td>
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<tr>
<td>( R^2 )</td>
<td>(0.022)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.400)</td>
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<td></td>
<td>(0.236)</td>
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<tr>
<td><strong>Eff S</strong></td>
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<tr>
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<td>-0.047</td>
<td>0.060</td>
<td>-0.470</td>
<td>0.072</td>
<td>-0.317</td>
<td>0.896</td>
<td>-0.224</td>
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<td>(&lt;0.001)**</td>
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<td>(&lt;0.001)**</td>
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<tr>
<td>( R^2 )</td>
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<td>(0.186)</td>
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<td></td>
<td>(0.811)</td>
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<tr>
<td><strong>Resp</strong></td>
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<tr>
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<td>0.155</td>
<td>0.078</td>
<td>-0.445</td>
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<td>0.057</td>
<td>-0.289</td>
<td>0.820</td>
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<tr>
<td>( R^2 )</td>
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<td><strong>Pimpt</strong></td>
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<tr>
<td>Coffee</td>
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<td>-0.976</td>
<td>0.140</td>
<td>-0.461</td>
<td>-0.452</td>
<td>0.196</td>
<td>-0.521</td>
<td>0.786</td>
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<td>( p-value )</td>
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<td>(&lt;0.001)**</td>
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<td>( R^2 )</td>
<td>(0.332)</td>
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<td></td>
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<td>(0.484)</td>
<td></td>
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<td>(0.705)</td>
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</table>

The hidden liquidity measure is \( AskHDC_{i,t} \) is a fraction of sell hidden order volume and trades volume; \( BidHDC_{i,t} \) is a fraction of buy hidden order volume and trades volume. \( MktQuality_{i,t} \) is a placeholder denoting: \( Spread \) is quoted spread; \( EffSpread \) is the effective spread; \( ReSpread \) is the realized spread; \( Pimpt \) is the price impact. \( MarketConcentration \) is the function of a number of traders and their respective volume of the total quote volume in LOB.
VAR & IRF (01)

Impulse response function of $\widehat{QR}$, $\widehat{TD}$, $\widehat{HDV}$ and $\widehat{TV}$ on E-mini S&P500 for one week to maturity.
**VAR & IRF (02)**

**Impulse response function of** $\hat{QR}$, $\hat{TD}$, $\hat{HDV}$ and $\hat{TV}$ **on E-mini S&P500 for six week to maturity.**

<table>
<thead>
<tr>
<th>Milliseconds (10^2)</th>
<th>Milliseconds (10^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>QR</td>
</tr>
<tr>
<td></td>
<td>TD</td>
</tr>
<tr>
<td></td>
<td>HDV</td>
</tr>
<tr>
<td></td>
<td>TV</td>
</tr>
</tbody>
</table>

**Resp. of Quote Return to QR, TD, HDV and TV**

- $QR$
- $TD$
- $HDV$
- $TV$

**Resp. of Trade Direction to QR, TD, HDV and TV**

- $QR$
- $TD$
- $HDV$
- $TV$

**Resp. of Hidden Order Volume to QR, TD, HDV and TV**

- $QR$
- $TD$
- $HDV$
- $TV$

**Resp. of Trade Volume to QR, TD, HDV and TV**

- $QR$
- $TD$
- $HDV$
- $TV$
Impulse response function of $\hat{QR}$, $\hat{TD}$, $\hat{HDV}$ and $\hat{TV}$ on E-mini S&P500 for 12 week to maturity.
Impulse response function of $\tilde{QR}$, $\tilde{TD}$, $\tilde{HDV}$ and $\tilde{TV}$ on E-mini S&P500 for 18 week to maturity.
We develop a detection algorithm to detect hidden order in limit order book using publicly available data.

The result of price impact shows that traders who using invisible order strategy are a strategies trader who trades based on their privileged information which they know when to submit their hidden orders.

The empirical results show that hidden liquidity positively improves the traditional yardsticks of market quality except inside quote spread.

Our evidence is more favorable to the notion that hidden order activity improves market quality whether high or low-frequency trading condition for E-mini S&P500 index future market. The hidden order activity favor to trading volume as the $HDV$ is increased when the $TV$ increase.
Thanks!


